Exercice (3)

On considère un cycle d'Ericsson composé des transformations réversibles suivantes:

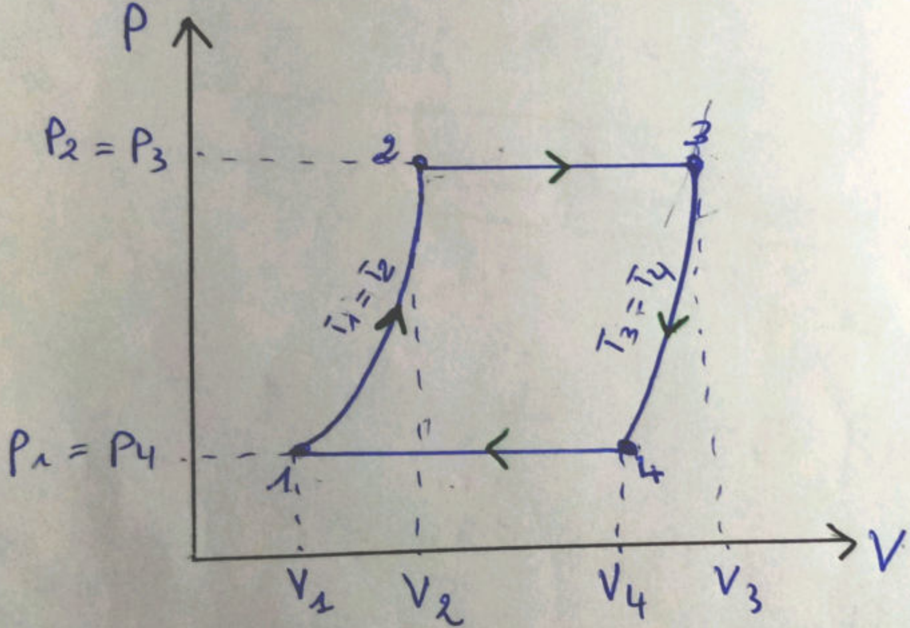
$1 \to 2 \Rightarrow$ compression isotherme $(T_1 = T_2)$

$2 \to 3 \Rightarrow$ Echauffement ~~Détente~~ isobare $(P_2 = P_3)$

$3 \to 4 \Rightarrow$ Détente isotherme $T_3 = T_4$ $(T_3 > T_1)$

$4 \to 1 \Rightarrow$ Refroidissement ~~compression~~ isobare $(P_4 = P_1$ telleque $P_1 < P_2)$

1) Digramme (P, V)



2) Les travaux et quantités de chaleur lors du cycle.

a) * chemin $1 \to 2$: compression isotherme. $(T_1 = T_2)$

$$PV = nRT_1 \Rightarrow \boxed{P = \frac{nRT_1}{V}}$$

- $\delta W_{1\to 2} = -P\,dV \Rightarrow W_{1\to 2} = -\int_{V_1}^{V_2} \frac{nRT_1}{V}\,dV$

$$\Rightarrow W_{1\to 2} = -nRT_1 \left[\ln\left(\frac{V_2}{V_1}\right)\right]$$

Etat 1 : $P_1 V_1 = nRT_1$

Etat 2 : $P_2 V_2 = nRT_2 = nRT_1$

$\Rightarrow P_1 V_1 = P_2 V_2 \Rightarrow \boxed{\dfrac{V_2}{V_1} = \dfrac{P_1}{P_2}}$

$$W_{1\to 2} = -nRT_1 \ln\left(\frac{V_2}{V_1}\right) = -nRT_1 \ln\left(\frac{P_1}{P_2}\right)$$

$$\boxed{W_{1\to 2} = nRT_1 \ln\left(\frac{P_2}{P_1}\right)}$$

- $$\delta Q_{1\to 2} = \underset{(T=cte)}{C_V \cancel{dT}} + p\,dV = -\delta W_{1\to 2}$$

$$\Rightarrow Q_{1\to 2} = -W_{1\to 2} = -nRT_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$\boxed{Q_{1\to 2} = nRT_1 \ln\left(\frac{P_1}{P_2}\right)}$$

b)

* Chemin $2 \to 3$: Echauffement isobare. $(P_2 = P_3)$

- $\delta W_{2\to 3} = -p\,dV$; $P = P_2 = P_3 \to$ isobare. ($P =$ cte)

$$W_{2\to 3} = -P_2 \int_{V_2}^{V_3} dV = -P_2 \left[V_3 - V_2\right]$$

Etat 2, $P_2 V_2 = nRT_2 = nRT_1$

Etat 3, $P_3 V_3 = nRT_3$

$\rightarrow \begin{cases} \boxed{V_2 = \dfrac{nRT_1}{P_2}} \\ V_3 = \dfrac{nRT_3}{P_3} \quad (P_3 = P_2) \end{cases}$

$$\boxed{V_3 = \frac{nRT_3}{P_2}}$$

$$W_{2\to 3} = -P_2 \left[\frac{nRT_3}{P_2} - \frac{nRT_1}{P_2}\right]$$

$$\boxed{W_{2\to 3} = -nR\left[T_3 - T_1\right]}$$

• $\delta Q_{2\to 3} = C_p\, dT - \underbrace{V\,dp}_{(p = cte)} = C_p\, dT.$

$$Q_{2\to 3} = C_p \int_{T_2 = T_1}^{T_3} dT = C_p\left[T_3 - T_1\right]$$

$$\begin{cases} C_p - C_v = nR \\ \gamma = \dfrac{C_p}{C_v} \end{cases} \Rightarrow \quad C_p = \frac{nR\gamma}{\gamma - 1}$$

$$\Rightarrow \quad Q_{2\to 3} = C_p\left[T_3 - T_1\right] \Rightarrow \boxed{Q_{23} = \frac{nR\gamma}{\gamma - 1}\left[T_3 - T_1\right]}$$

c/ * Chemin $3 \to 4$: Détente isotherme $(T_3 = T_4) \to (T_3 > T_1)$

$$PV = nRT_3 \Rightarrow P = \frac{nRT_3}{V}$$

$$\delta W_{3\to 4} = -p\,dV \quad \Rightarrow \quad W_{3\to 4} = -\int nRT_3 \frac{dV}{V}$$

$$\Rightarrow W_{3\to 4} = -nRT_3 \int_{V_3}^{V_4} \frac{dV}{V} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right)$$

Etat 3 : $\quad P_3 V_3 = nRT_3 \quad (P_3 = P_2) \Rightarrow P_2 V_3 = nRT_3$

Etat 4 : $\quad P_4 V_4 = nRT_4 \quad (P_4 = P_1$ et $T_4 = T_3)$

$$\Rightarrow P_1 V_4 = nRT_3 \Rightarrow V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow \quad V_3 = \frac{nRT_3}{P_2} \quad \text{et} \quad V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow \quad V_4 = \frac{V_3 P_2}{P_1} \Rightarrow \boxed{\frac{V_4}{V_3} = \frac{P_2}{P_1}}$$

$$W_{3\to 4} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right) = -nRT_3 \ln\left(\frac{P_2}{P_1}\right)$$

$$\boxed{W_{3\to 4} = nRT_3 \ln\left(\frac{P_1}{P_2}\right)}$$

- $\delta Q_{2\to 3} = C_p\, dT - \underset{(p = cte)}{\cancel{V\,dp}} = C_p\, dT.$

$$Q_{2\to 3} = C_p \int_{T_2 = T_1}^{T_3} dT = C_p\left[T_3 - T_1\right]$$

$$\begin{cases} C_p - C_v = nR \\ \gamma = \dfrac{C_p}{C_v} \end{cases} \Rightarrow C_p = \frac{nR\gamma}{\gamma - 1}$$

$$\Rightarrow Q_{2\to 3} = C_p\left[T_3 - T_1\right] \Rightarrow \boxed{Q_{23} = \frac{nR\gamma}{\gamma - 1}\left[T_3 - T_1\right]}$$

c/ * Chemin $3 \to 4$: Détente isotherme $(T_3 = T_4) \to (T_3 > T_1)$

$$PV = nRT_3 \Rightarrow P = \frac{nRT_3}{V}$$

$$\delta W_{3\to 4} = -p\,dV \Rightarrow W_{3\to 4} = -\int nRT_3 \frac{dV}{V}$$

$$\Rightarrow W_{3\to 4} = -nRT_3 \int_{V_3}^{V_4} \frac{dV}{V} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right)$$

Etat 3 : $P_3 V_3 = nRT_3 \quad (P_3 = P_2) \Rightarrow P_2 V_3 = nRT_3$

Etat 4 : $P_4 V_4 = nRT_4 \quad (P_4 = P_1 \text{ et } T_4 = T_3)$

$$\Rightarrow P_1 V_4 = nRT_3 \Rightarrow V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow V_3 = \frac{nRT_3}{P_2} \text{ et } V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow V_4 = \frac{V_3 P_2}{P_1} \Rightarrow \boxed{\frac{V_4}{V_3} = \frac{P_2}{P_1}}$$

$$W_{3\to 4} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right) = -nRT_3 \ln\left(\frac{P_2}{P_1}\right)$$

$$\boxed{W_{3\to 4} = nRT_3 \ln\left(\frac{P_1}{P_2}\right)}$$

- $\delta Q_{2\to 3} = C_p\, dT - \underbrace{V\,dP}_{(P=cte)} = C_p\, dT$.

$$Q_{2\to 3} = C_p \int_{T_2=T_1}^{T_3} dT = C_p\,[T_3 - T_1]$$

$$\begin{cases} C_p - C_v = nR \\ \gamma = \dfrac{C_p}{C_v} \end{cases} \Rightarrow C_p = \frac{nR\gamma}{\gamma-1}$$

$$\Rightarrow Q_{2\to 3} = C_p\,[T_3 - T_1] \Rightarrow \boxed{Q_{23} = \frac{nR\gamma}{\gamma-1}[T_3 - T_1]}$$

c/ * Chemin $3\to 4$: Détente isotherme $(T_3 = T_4) \to (T_3 > T_1)$

$$PV = nRT_3 \Rightarrow P = \frac{nRT_3}{V}$$

$$\delta W_{3\to 4} = -P\,dV \Rightarrow W_{3\to 4} = -\int nRT_3 \frac{dV}{V}$$

$$\Rightarrow W_{3\to 4} = -nRT_3 \int_{V_3}^{V_4} \frac{dV}{V} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right)$$

Etat 3 : $P_3V_3 = nRT_3 \quad (P_3 = P_2) \Rightarrow P_2V_3 = nRT_3$

Etat 4 : $P_4V_4 = nRT_4 \quad (P_4 = P_1 \text{ et } T_4 = T_3)$

$$\Rightarrow P_1V_4 = nRT_3 \Rightarrow V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow V_3 = \frac{nRT_3}{P_2} \text{ et } V_4 = \frac{nRT_3}{P_1}$$

$$\Rightarrow V_4 = \frac{V_3P_2}{P_1} \Rightarrow \boxed{\frac{V_4}{V_3} = \frac{P_2}{P_1}}$$

$$W_{3\to 4} = -nRT_3 \ln\left(\frac{V_4}{V_3}\right) = -nRT_3 \ln\left(\frac{P_2}{P_1}\right)$$

$$\boxed{W_{3\to 4} = nRT_3 \ln\left(\frac{P_1}{P_2}\right)}$$

- $\delta Q_{3\to4} = \underset{(T=cte)}{C_v \cancel{dT}^{0}} + p\,dV = -\delta W_{3\to4}$

$$Q_{3\to4} = -W_{3\to4} = -nRT_3 \ln\left(\frac{P_1}{P_2}\right)$$

$$\boxed{Q_{3\to4} = nRT_3 \ln\left(\frac{P_2}{P_1}\right)}$$

d) chemin $4 \longrightarrow 1$, Refroidissement isobare. $(P_4 = P_1)$

$$\delta W_{4\to1} = -P_1\,dV \Rightarrow W_{4\to1} = -P_1 \int_{V_4}^{V_1} dV$$

$$W_{4\to1} = -P_1\left[V_1 - V_4\right]$$

Etat 4 : $P_4V_4 = nRT_4 \Rightarrow P_1V_4 = nRT_3 \Rightarrow \boxed{V_4 = \frac{nRT_3}{P_1}}$

" 1 : $P_1V_1 = nRT_1 \Rightarrow$ Donc $\boxed{V_1 = \frac{nRT_1}{P_1}}$

$$W_{4\to1} = -P_1\left[\frac{nRT_1}{P_1} - \frac{nRT_3}{P_1}\right] = -nR\left[T_1 - T_3\right]$$

$$\boxed{W_{4\to1} = -nR\left[T_1 - T_3\right]}$$

- $\delta Q_{4\to1} = C_v\,dT - \cancel{V\,dP} = C_v\,dT$

$$Q_{4\to1} = C_v \int_{T_4}^{T_1} dT = C_v\left[T_1 - T_4\right] = C_v\left(T_1 - T_3\right)$$

$$Q_{4\to1} = C_v\left[T_1 - T_3\right] = \frac{nR}{\gamma-1}\left[T_1 - T_3\right]$$

$$\boxed{Q_{4\to1} = \frac{nR}{\gamma-1}\left[T_1 - T_3\right]}$$

3) a- La quantité de chaleur reçu par le système au cours du cycle,

$$Q_{reçu} = Q_{1\to2} + Q_{2\to3} = nRT_1 \ln\left(\frac{P_1}{P_2}\right) + \frac{nR\gamma}{\gamma-1}\left[T_3 - T_1\right]$$

3) b- La quantité de chaleur cédée par le système au cour du cycle.

$$Q_{cédée} = Q_{3\to 4} + Q_{4\to 1}$$

$$= nRT_3 \ln\left(\frac{P_2}{P_1}\right) + \frac{nR}{\gamma - 1}\left[T_1 - T_3\right]$$

c- Le travail échangé avec le milieu extérieur.

$$\Delta U = W + Q \Big\} \text{ 1ère principe de thermody.}$$

Lors du cycle $\Delta U_{cycle} = 0 \Leftrightarrow \begin{cases} W + Q = 0 \\ Q = Q_{reçu} + Q_{cédé} \end{cases}$

$$W = -Q = -\left(Q_{reçu} + Q_{cédée}\right)$$

$$W = -\left[nRT_3 \ln\left(\frac{P_2}{P_1}\right) + \frac{nR}{\gamma - 1}\left[T_1 - T_3\right]\right]$$

$$\boxed{W = nRT_3 \ln\left(\frac{P_1}{P_2}\right) + \frac{nR}{\gamma - 1}\left(T_3 - T_1\right)}$$

4) La variation d'entropie qui accompagne chaque transformation de ce cycle.

- $\Delta S_{1\to 2} = \dfrac{Q_{1\to 2}}{T_1} = nR \ln\left(\dfrac{P_1}{P_2}\right)$
- $\Delta S_{3\to 4} = \dfrac{Q_{3\to 4}}{T_3} = nR \ln\left(\dfrac{P_2}{P_1}\right) = -nR\ln\left(\dfrac{P_1}{P_2}\right)$
- $dS_{2\to 3} = \dfrac{\delta Q_{2\to 3}}{T} = C_v \dfrac{dT}{T} \Rightarrow \Delta S_{2\to 3} = C_v \displaystyle\int_{T_2=T_1}^{T_3} \dfrac{dT}{T}$

$$\Delta S_{2\to 3} = \frac{nR}{\gamma - 1} \ln\left(\frac{T_3}{T_1}\right)$$

- $dS_{4\to 1} = \dfrac{\delta Q_{4\to 1}}{T} = C_v \dfrac{dT}{T} \Rightarrow \Delta S_{4\to 1} = C_v \displaystyle\int_{T_3=T_4}^{T_1} \dfrac{dT}{T}$

$$\Delta S_{4\to 1} = \frac{nR}{\gamma - 1} \ln\left(\frac{T_1}{T_3}\right) = -\frac{nR}{\gamma - 1} \ln\left(\frac{T_3}{T_1}\right)$$